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Extension of Wheeler–Feynman quantum theory to the relativistic domain II. Emission processes

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Abstract. In paper I we examined from the standpoint of the S matrix that inside a light tight box the photon propagator D_F may be replaced by its real part $\bar{D}$, without change in the quantum generalization of the classical absorber theory of Wheeler and Feynman.

In this paper, we first examine the classical radiation field, clarifying some concepts and definitions, then apply the results of I to derive the usual expressions for the real photon processes of conventional quantum electrodynamics.

1. Classical radiation field

Suppose we solve the wave equation for a point charged particle in arbitrary motion, retaining the time symmetry for as long as possible. We can construct two fields which are antisymmetric and symmetric under T inversion, respectively

$$\bar{A} = \tfrac{1}{2}(A^{\text{ret}} + A^{\text{adv}}) \tag{1}$$

$$A = \tfrac{1}{2}(A^{\text{ret}} - A^{\text{adv}}). \tag{2}$$

The first field is a solution of the inhomogeneous equation while the second field is source free. This field (A) has been called ($\frac{1}{2}$) the radiation field by Dirac (1938), but our usual notion of such a field is a long range ($1/R$) acceleration (a^{μ}) field at retarded infinity. But in this region the two definitions are the same (ignoring the $\frac{1}{2}$), as $A^{\text{adv}} = 0$. However, Dirac's definition has the advantage that if $a^{\mu} = 0$ the radiation field vanishes everywhere not just at retarded infinity†. Although A is source free, we can envisage both A and $\bar{A}$ as spherical waves converging on the particle from advanced infinity, re-emerging and expanding to retarded infinity, the only difference being that $\bar{A}$ suffers a phase change of π at the particle world line (see figure 1). If we now superpose both fields, we can conform to the various boundary conditions by arranging them either in phase at

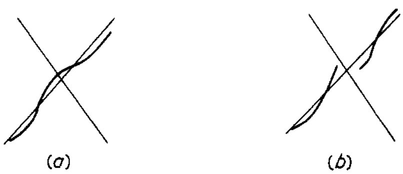

Figure 1. (*a*) The field A; (*b*) the field $\bar{A}$.

† Because it is a solution of $\Box A = 0$.

$t = -\infty$, giving advanced fields only, or π out of phase, at $t = -\infty$, giving retarded fields only (see figure 2). Moreover, A can be invariantly decomposed into positive $(A+)$ and negative $(A-)$ frequencies, which may be separately phased, to give retarded $+$ and advanced $-$ or vice versa.

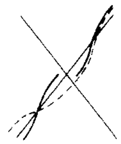

Figure 2. Interference.

Consider the decomposition

$$A_{\text{ret}} = A + \bar{A}. \tag{3}$$

It is often remarked that A gives the far field and $\bar{A}$ the near ($1/R^2$, or velocity) field. But this is misleading, because A also has an advanced component, and its retarded component is only $\frac{1}{2}$ the full value. $\bar{A}$ also has $1/R$ fields, which give the usual full retarded (or advanced) far field by interference with A. However, for uniform motion ($a^\mu = 0$), the far fields vanish, so A vanishes everywhere. Therefore $\bar{A}$ will describe the velocity fields only. We conclude that the velocity fields are time symmetric†. When we examine the action of these fields on the source particle, the $\bar{A}$ field gives rise to the (divergent) selfenergy of the particle, whilst A gives the finite radiative damping force, but both fields carry away the radiation which is generated.

If we now enclose the system in a light tight box outside of which there is no source free radiation from infinity, then the *total* A field vanishes throughout by the usual absorber argument. Thus the A field for a particular particle is due to the $\bar{A}$ field of all the other particles collectively interfering, and the converging–diverging A wave discussed above can be regarded as the advanced response of the absorber to the particle motions (see Wheeler and Feynman 1945 for a careful discussion of this). The vanishing of the field A means that it can be removed from our description of the system without changing the results for a light tight box. But the removal of all the free field excitation enables us to dispense with the field altogether as an independent mechanical system, for it can only mediate the interaction of the particles (through $\bar{A}$), and never remove energy and momentum to infinity (through A). We thus arrive at an action-at-a-distance formalism. This is the usual Wheeler–Feynman theory.

2. Real and virtual photons

When we quantize the free electromagnetic field, we build up a Fock space out of states containing all numbers of photons. These photons obey the relation $k^2 = 0$, and, by the uncertainty principle, have an infinite lifetime. When the field is coupled to its sources, we allow for photons to be created and annihilated. If a photon is created at $t = 0$, and destroyed at $t = T$, we expect that it will lie off the energy shell (ie that $k^2 \neq 0$)

† However, we cannot detect the advanced (precursor) signals in the velocity field (or Coulomb) case, because of the uncertainty principle (see below).

for finite T. We say that such photons are virtual. However, this simple picture can be very misleading and confusing.

To understand this, we recall the property proved in Davies (1971a, to be referred to as I) for the S matrix†

$$\langle 0|P\exp(-\mathrm{i}\textstyle\int J(x)A(x)\,\mathrm{d}x)|0\rangle = P\exp\left(\tfrac{1}{2}\mathrm{i}\displaystyle\int\int J(x)D_{\mathrm{F}}(x-y)J(y)\,\mathrm{d}x\,\mathrm{d}y\right)$$

where $J(x) = \Sigma_i j_{(i)}(x)$, i running over all species of particles, and $|0\rangle$ refers to the photon vacuum only. The expression on the right hand side of (4) may be used to calculate the contributions from all Feynman graphs with internal photon lines only. By taking the vacuum photon state we indicate that the system has no real photons at $t = \pm\infty$. However, let us examine the photon propagator D_{F} in detail. A Fourier decomposition gives

$$D_{\mathrm{F}}(x) = \frac{1}{(2\pi)^4}\int\left(\frac{\mathrm{PP}}{k^2} - \mathrm{i}\pi\delta(k^2)\right)\mathrm{e}^{\mathrm{i}kx}\,\mathrm{d}k \tag{5}$$

$$= \bar{D} + D_1 \tag{6}$$

where PP is the principal part. The $\bar{D}$ part (bound field) leads to the real principal part term which describes virtual photons ($k^2 \neq 0$), whilst the imaginary D_1 (free field) describes photons with $k^2 = 0$, that is, real photons, through the δ function term. This closely parallels the classical situation, that is, the virtual photons (time symmetric, bound field) give rise to the near field, because of the finite lifetime of the virtual photons, while the real photons can escape to infinity as the far field. But how do we reconcile the notion of a real photon as an *internal* line in the Feynman diagram with the uncertainty principle? In other words, how can a real photon, which ought to have an infinite lifetime, be emitted and reabsorbed, as described by the right hand side of (4)?

The paradox can be resolved by appealing to the classical theory. Just as we can never separate the A and $\bar{A}$ fields, and both of them carry away radiation when $a^\mu \neq 0$, so the virtual photons continually interfere with the real photons when we have the quantum analogue of acceleration (ie energy available for the transition). This interference leads to both real and virtual photons carrying energy to retarded infinity. It is the virtual photons with very small k^2 that provide the long range field‡. For negative times, these virtual photons just cancel the advanced field of the real photons, so that for a positive energy source we only have disturbances propagated into the future. We can now explain physically how a real photon can be absorbed. We may say that it has existed for an infinite time, but the virtual photons have cancelled its advanced effects by interference (see figure 2). If we take the lowest order term in e^2 in the expansion of (4) we obtain a matrix element with a real part (including $\bar{D}$) and an imaginary part (including D_1). The real part gives rise to the selfenergy and level shift, whilst the imaginary part gives the level width, or transition rate for real photon emission, in analogy to the classical case.

The whole real/virtual terminology is confused as many authors, such as Feynman, call a photon virtual merely if it has a finite lifetime, that is, it is an internal line on a Feynman graph. Thus in his book 'Theory of Fundamental Processes' Feynman draws a diagram like figure 3 and remarks

† We suppress the vector indices for simplicity.

‡ Without appreciating that virtual photons can contribute to radiative effects and produce a far field also, the Wheeler–Feynman notion of having virtual photons only is incomprehensible.

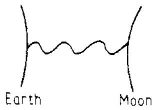

Figure 3.

"In a sense every real photon is actually virtual if one looks over sufficiently long time scales". But of course, he does not mean that we should use only the $1/k^2$ (or $\bar{D}$) part of the propagator, for this would give a ($\frac{1}{2}$ advanced + $\frac{1}{2}$ retarded) field.

If there was no energy source at 'earth' so that we could not pick up the $k^2 = 0$ in the δ function anyway, then this would be correct. However, we cannot separate the advanced and retarded signals because of the uncertainty principle. To see this, we appreciate that when no energy source is available, the time ΔT required for the emission process is related to the frequency of the photon by $\Delta T \sim 1/\omega$. That is, in the wave zone of the source, we are completely unsure of the order of emission and absorption. This order is only well defined in the far zone, but the far field vanishes here. When there is an energy source, we also have a real photon contribution from the δ function, and this cancels the advanced signal and reinforces the retarded. The energy provided also enables us to remove the uncertainty principle restriction as not being able to measure the order of emission and absorption, so that we may certainly say that a signal has been sent from earth to moon, rather than vice versa.

Now we come to the crucial point. If we put the system in a light tight box, we know from the classical theory that the free field vanishes. So we may say that *there can be no real photons inside a box if there are none outside.* Then the $\delta(k^2)$ or D_1 part of the propagator can be omitted. We may express this as the following theorem:

Theorem 2(i) Inside a light tight box all photons are virtual. This is, of course, a tautology by Feynman's definition, but not by ours, for it implies:

Theorem 2(ii) Inside a light tight box the photon propagator (D_F) can be replaced by its real part ($\bar{D}$).

Although physically reasonable from the above argument, to prove this we recall the discussion in I.

Consider the expression $\langle 0|SS^\dagger|0\rangle$. Using equation (4) and unitarity, we have $\langle 0|SS^\dagger|0\rangle = 1$ so

$$\sum_{n\geq 1}\sum_{\psi}|\langle 0|P\exp(-\mathrm{i}\textstyle\int J(x)A(x)\,\mathrm{d}x)|n\rangle|^2$$

$$= 1 - \sum_{\psi}\left|\left\langle P\exp\left(\tfrac{1}{2}\mathrm{i}\int\int J(x)(\bar{D}+D_1)J(y)\,\mathrm{d}x\,\mathrm{d}y\right)\right\rangle\right|^2. \qquad (7)$$

The ψ indicates summation over fermion states. The lefthand side of (7) is just the total transition probability for the emission of $1, 2, 3, \ldots$ photons (to all orders). We wish to demonstrate that when the lefthand side vanishes, we may remove the imaginary D_1 (real photon) term from the righthand side. Now the emission term will vanish for certain restrictive types of fermion states ψ. One such set of states would be the ground states of all atoms, or the set of all free one particle states. In these cases it is well known that we may use only the $\bar{D}$ part of the propagator as the photons can never get on the energy shell because of the conservation laws. This is the case when calculating the usual van der Waal's force between ground state atoms, for instance. For our purposes, however, we are interested in the states ψ corresponding to a light tight box. In this case it is clear that real photons can occur in some terms in S without violating any conservation laws, so that individual D_1 terms will contribute. Nevertheless, when we take the totality of such terms, we can show that their contributions all cancel by interference, so that, provided we consider all the particles of the system (including the box), these real photons may be omitted.

Now for the subset of states ψ which preclude emission processes, the submatrix given by the last term of (7) is obviously unitary. If the currents J are classical this leads to the immediate requirement that the imaginary term in the exponential must vanish. For nonclassical currents, the removal of this term renders the integrand hermitean, though because of the P operator, even this matrix is not manifestly unitary as in the classical case. But we know that for our light tight box states this term must give unity, so that the contribution from the D_1 term, as well as unwanted terms arising from P should vanish for these particular states, although it is just possible that they will cancel.

To exclude this possibility, and to provide a straightforward demonstration that the D_1 terms do vanish in this case, we can appeal to the property of the matrix elements noted by Feynman (1950). The property is based on the physical notion that when a real photon is exchanged, the emission and absorption processes may be considered as independent. Mathematically, this is expressed in our ability to factorize the matrix element with an internal photon line into the product of two matrices, one depending only on the emission variables, the other on the absorption variables. Clearly then, if the emission processes have vanishing probability, so do the exchange processes. That is, if a real photon cannot be emitted, it obviously cannot be emitted and reabsorbed again. In the case of the light tight box, it is clear that if we consider a small region, a real photon can be emitted and the argument does not apply, but when we include the whole box the argument is valid. This is made clearer if we consider the selfenergy graph (figure 4). If we factorize S so that the emission and absorption events at A and B are independent, then there is only a real photon contribution to this graph if the emission process in figure 5 has nonzero probability. If figure 5 is excluded (eg for a ground state atom) then so is the real photon exchange, A $\rightarrow$ B. This is none other than the familiar unitarity property that the D_1 (imaginary) part of the selfenergy matrix element is equal to the first order emission probability, so that for the ground state the level width is zero.

Figure 4. Selfenergy

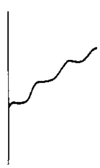

Figure 5. Emission.

Now in the case of a light tight box the fermion line in figure 4 represents, not just one current, but a very large number ($J = \Sigma_i j_{(i)}$). In this case the selfenergy graph represents all conceivable one photon exchange processes inside the box, and it is only when we add all these matrix elements together that they cancel by interference to give zero. The same considerations apply for any number of internal photon lines.

In the next section we shall perform the above mentioned factorization of S in detail.

3. Real photon matrix elements

Knowing the matrix elements for processes with internal photon lines we can easily obtain the matrix elements for emission processes from unitarity considerations. Let us consider the case of classical currents. Our fundamental form for the S matrix for a single current becomes

$$S = \exp\left(\tfrac{1}{2}\mathrm{i} \int\int J_{(i)}(x)(\bar{D}(x-y)+D_1(x-y))J_{(i)}(y)\, \mathrm{d}x\, \mathrm{d}y\right) \tag{8}$$

so that

$$SS^{\dagger} = \mathrm{e}^{-\bar{n}} \neq 1$$

where

$$\bar{n} = -\mathrm{i} \int\int J_{(i)}(x)D_{+}(x-y)J_{(i)}(y)\, \mathrm{d}x\, \mathrm{d}y \tag{9}$$

which is real and positive, and in Maxwell theory can be identified with the average number of emitted photons. S is not unitary, because the action in (8) is complex. If however, we had considered all currents in a light tight box, then S would be unitary because of the absorber condition

$$\sum_i \sum_j \int\int J_{(i)}(x)D_{+}(x-y)J_{(j)}(y)\, \mathrm{d}x\, \mathrm{d}y = 0. \tag{10}$$

By extracting a single current i from the double summation in (10), we are not taking into account interactions with the walls of the box, which are regarded physically in the region of i as real photon emission processes. Hence S cannot be unitary. Evidently these omitted processes occur with probability $1-\mathrm{e}^{-\bar{n}}$. This may be written as a perturbation expansion

$$1-\mathrm{e}^{-\bar{n}} = \sum_{n=1}^{\infty} \mathrm{e}^{-\bar{n}}\frac{\bar{n}^n}{n!}. \tag{11}$$

This is the familiar Poisson distribution often discussed in connection with the infrared divergence. Each successive term on the righthand side of (10) may be interpreted as the emission probability of 1, 2, 3, ... photons.

In his paper Feynman (1950) showed another way to construct matrix elements for real photon processes knowing the matrix elements for virtual photons. Remembering his definitions this means that we may convert information about internal real photon lines to information about external real photon lines by noticing that what is an external line in a small region can become an internal line when larger dimensions are considered. We know that what happens to a real photon a million miles away does not affect its emission process in the laboratory. This physical decoupling of emission and absorption processes is reflected in the properties of the S matrix.

Suppose we have two currents i and j separated by a great distance so that they may be considered distinguishable. We write the S matrix as a product of three factors

$$\begin{aligned} P \exp&\left(\tfrac{1}{2}\mathrm{i} \int\int j_{(i)}(x)D_{\mathrm{F}}(x-y)j_{(i)}(y)\, \mathrm{d}x\, \mathrm{d}y\right) \\ &\times \exp\left(\tfrac{1}{2}\mathrm{i} \int\int j_{(j)}(x)D_{\mathrm{F}}(x-y)j_{(j)}(y)\, \mathrm{d}x\, \mathrm{d}y\right) \\ &\times \exp\left(\tfrac{1}{2}\mathrm{i} \int\int j_{(i)}(x)D_{\mathrm{F}}(x-y)j_{(j)}(y)\, \mathrm{d}x\, \mathrm{d}y\right). \end{aligned} \tag{12}$$

The first two factors of (12) contain operators referring respectively to the currents i and j only. Expanding this term to lowest order in e^2, and abbreviating the first two terms in an obvious way, we have (reinstating the vector indices for clarity)

$$P\,\mathrm{e}^{\mathrm{i}\alpha_{(i)}}\,\mathrm{e}^{\mathrm{i}\alpha_{(j)}}\mathrm{i}\int\int j^{\mu}_{(i)}(x)D_{\mathrm{F}}(x-y)g_{\mu\nu}j^{\nu}_{(j)}(y)\,\mathrm{d}x\,\mathrm{d}y. \tag{13}$$

Expanding the D_{F} function in Fourier components gives

$$D_{\mathrm{F}}(x-y) = \mathrm{i}\sum_{\boldsymbol{k}}\frac{\exp\{\mathrm{i}\boldsymbol{k}\,.\,(\boldsymbol{x}-\boldsymbol{y})-\mathrm{i}\omega|x^0-y^0|\}}{2\omega\Omega}. \tag{14}$$

Now i and j are well separated physically, so we may decide that, say i is emitting radiation and j receiving it. Then $x^0 < y^0$ and we may remove the modulus sign from the exponent of (14). This fact enables us to factorize (14) into two parts

$$D_{\mathrm{F}}(x-y) \to \mathrm{i}\sum_{\boldsymbol{k}}\left(\frac{\exp(\mathrm{i}\boldsymbol{k}\,.\,\boldsymbol{x}+\mathrm{i}\omega x^0)}{(2\omega\Omega)^{1/2}}\,\frac{\exp(-\mathrm{i}\boldsymbol{k}\,.\,\boldsymbol{y}-\mathrm{i}\omega y^0)}{(2\omega\Omega)^{1/2}}\right). \tag{15}$$

We can interpret the sum over $\boldsymbol{k}$ as a sum over intermediate states for which we must specify a momentum $\boldsymbol{k}$. Furthermore, we may factorize the $g_{\mu\nu}$

$$g_{\mu\nu} = \sum_{\lambda=1}^{4} e^{\lambda}_{\mu}e^{\lambda}_{\nu} \tag{16}$$

where the e are othornomal basis vectors, so that these states must also be specified by a polarization index λ. We are thus led to invent photons of momentum $\boldsymbol{k}$ and polarization λ in trying to represent the effect of the past (the current i) on the future (the current j).

We may see this clearly by returning to (14). Removal of the modulus sign changes D_{F} to $-D_-$. Inserting this in (13) gives

$$-P\,\mathrm{e}^{\mathrm{i}\alpha_{(i)}}\,\mathrm{e}^{\mathrm{i}\alpha_{(j)}}\mathrm{i}\int\int j^{\mu}_{(i)}(x)D_-(x-y)g_{\mu\nu}j^{\nu}_{(j)}(y)\,\mathrm{d}x\,\mathrm{d}y. \tag{17}$$

Now we may *formally* invent operators $A_\mu(x)$, $A_\nu(y)$ such that

$$\langle 0|A_\nu(y)A_\mu(x)|0\rangle = \mathrm{i}D_-(x-y)g_{\mu\nu} \tag{18}$$

which is, of course, the usual rule for the photon operators. The lefthand side of (18) factorizes immediately

$$\langle 0|A_\nu(y)A_\mu(x)|0\rangle = \sum_{\boldsymbol{k},\lambda}\langle 0|A_\nu(y)|1_{\boldsymbol{k}\lambda}\rangle\langle 1_{\boldsymbol{k}\lambda}|A_\mu(x)|0\rangle \tag{19}$$

where we have invented intermediate states containing one photon of momentum $\boldsymbol{k}$ and polarization λ.

Substituting (19) into (17) allows the separation

$$\sum_{\boldsymbol{k},\lambda}\left(\mathrm{i}P\,\mathrm{e}^{\mathrm{i}\alpha_{(i)}}\int j^{\mu}_{(i)}(x)\langle 1_{\boldsymbol{k}\lambda}|A_\mu(x)|0\rangle\,\mathrm{d}x\right)$$
$$\times\left(\mathrm{i}P\,\mathrm{e}^{\mathrm{i}\alpha_{(j)}}\int j^{\nu}_{(j)}(y)\langle 0|A_\nu(y)|1_{\boldsymbol{k}\lambda}\rangle\,\mathrm{d}y\right). \tag{20}$$

In (20) we may let the P operators act separately on the currents i and j as they are distinguishable. Expression (20) may be written symbolically as $S^i_1 S^j_1$, where the first factor depends only on the current i and the second only on j. To accomplish this we have had to invent a photon Hilbert space complete with operators which obey the usual rules of the electromagnetic field operators.

To investigate higher order processes, consider further terms in the expansion of (12). The nth order term is

$$\sum_{k_1,\lambda_1} \dots \sum_{k_n,\lambda_n} \frac{(-1)^n}{n!} P\, e^{i\alpha_{(i)}} e^{i\alpha_{(j)}} \int \dots \int j^{\mu}_{(i)}(x_1) \dots j^{\sigma}_{(i)}(x_n)$$
$$\times \langle 0|A_\nu(y_1)|1_{k_1\lambda_1}\rangle\langle 1_{k_1\lambda_1}|A_\mu(x_1)|0\rangle \dots \langle 0|A_\rho(y_n)|1_{k_n\lambda_n}\rangle\langle 1_{k_n\lambda_n}|A_\sigma(x_n)|0\rangle$$
$$\times j^{\nu}_{(j)}(y_1) \dots j^{\rho}_{(j)}(y_n)\, dx_1 \dots dx_n\, dy_1 \dots dy_n. \tag{21}$$

If we interpret each $\boldsymbol{k}, \lambda$ as a photon in the n fold summation of (21), then there are n ways of labelling the n different values of $\boldsymbol{k}$ in the n summations corresponding to n distinguishable photons, and this just cancels the $1/n!$ in (21). However, if $\boldsymbol{k}$ is the same for all n photons there is only one term and we would say that identical photon states contribute a statistical weight of only $1/n!$ to the scattering amplitude. This is the rule of Bose statistics. If there are r identical photons among them, these may be chosen in $n!/(n-r)!r!$ different ways. The remaining $(n-r)$ summations may be used to label the $\boldsymbol{k}$ values in $(n-r)!$ ways. We thus have a factor $n!/r!$ which cancels the $1/n!$ in (21) to give $1/r!$.

Extracting the emission matrix element from (21) as a factor then gives

$$\frac{(-i)^n}{(r!)^{1/2}} P\, e^{i\alpha_{(i)}} \int \dots \int j^{\mu}_{(i)}(x_1) \dots j^{\sigma}_{(i)}(x_n) \langle 1_{k_1\lambda_1}|A_\mu(x_1)|0\rangle$$
$$\dots \langle 1_{k_n\lambda_n}|A_\sigma(x_n)|0\rangle\, dx_1 \dots dx_n. \tag{22}$$

We can now write the product of matrix elements of A as a single matrix element using the usual rules of creation and annihilation operators. That is, (22) may be written

$$\frac{(-i)^n}{n!} P\, e^{i\alpha_{(i)}} \int \dots \int j^{\mu}_{(i)}(x_1) \dots j^{\sigma}_{(i)}(x_n) \langle \Phi_r| N_A(A_\mu(x_1) \dots A_\sigma(x_n))|0\rangle \tag{23}$$

where N_A is a normal ordering operator introduced to ensure that only one matrix element contributes, Φ_r denotes the final state of n photons with r identical, 0 denotes the photon vacuum. The A may be expanded in creation and annihilation operators

$$A_\mu(x) = \sum_{k} (a^\dagger_k f_\mu(k, x) + a_k f^*_\mu(k, x)) \tag{24}$$

where the f_μ are c number functions. These operators have the property

$$(a^\dagger_k)^r|0\rangle = (r!)^{1/2}|r\rangle. \tag{25}$$

Therefore, use of the matrix element in (23) gives rise to a permutation factor of $n!/r!$ as before because of the summation in (24). It also involves an additional $(r!)^{1/2}$ from (25) for the r identical photons. Thus, in making the change from (22) to (23) we must divide by a factor $n!/(r!)^{1/2}$, which gives the $1/n!$ in (23).

Expression (23) is clearly the matrix element of the nth term of the exponential operator

$$P \exp\left(\tfrac{1}{2}i \int \int j^{\mu}_{(i)}(x) D_F(x-y) j_{(i)\mu}(y)\, dx\, dy\right) N_A \exp\left(-i \int j^{\mu}_{(i)}(x) A_\mu(x)\, dx\right) \tag{26}$$

which is the usual S matrix for all electrodynamic processes including emission and absorption of real photons (see, for example, Akhiezer and Berestetskii 1965, p 302 for reduction of S to this form). Note that we require N_A to act only on the A, to ensure that we will obtain nonzero matrix elements between $|0\rangle$ and $|n\rangle$ only from the nth term in the expansion of (26).

If we wish to discuss processes involving both absorption and emission of photons, we may include three currents in the S matrix, and perform the same separation of emission and absorption as above, only extracting the other (absorption) factor from (22) for the incoming photons together with (23) for the outgoing photons. Or we could treat the incoming photons directly in a semiclassical way (see, for example, Hoyle and Narlikar 1969). In all cases we will arrive at an expression formally identical to a matrix element of terms in the expansion of (26). The discussion provides a nice illustration of how we can dissect Feynman graphs.

4. Irreversibility

Up to this point the discussion of radiative effects is perfectly symmetrical in time. This is reflected in the fact that the photon propagator D_F is symmetric under time reversal. Physically, we know that for a single photon exchange between two particles, the reversal of time direction merely interchanges the roles of emitter and absorber, and describes an equally probable physical situation. The process is shown in figure 6,

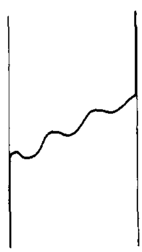

Figure 6.

which is invariant under rotation through 180°. This is a consequence of the well known time reversal invariance of electrodynamics. However, we know that certain radiative processes are irreversible. In Wheeler–Feynman theory this irreversibility arises naturally from the thermodynamical properties of the absorber, whereas it has no explanation in Maxwell theory. If we imagine particle B in figure 6 to be replaced by a large number of absorbing atoms initially in their ground states, then the energy from the emitter is gradually dispersed through these atoms in an irreversible way. The reverse situation, in which a large number of atoms make a coherent series of emissions is highly improbable (except by manipulation, as in the case of the maser). In the classical Wheeler–Feynman theory, we reach the retarded potential by adding the response field due to the total collection of absorbing particles which behave irreversibly in accordance with the laws of thermodynamics. Inside a box in thermal equilibrium coherence is destroyed and radiative processes behave reversibly. The universe is prevented from reaching this condition by the cosmological expansion.

In the quantum theory we also distinguish between positive and negative frequencies. In Maxwell theory, only positive frequency photons are allowed to propagate into the future, as the electromagnetic field has positive definite energy density. In Wheeler–Feynman theory, we could equally well subtract the quantity D_1 from $\bar{D}$ to arrive at D_F^* rather than D_F. This would correspond to negative frequencies being propagated forwards in time and would lead to a situation depicted by figure 7. It is important to realize that this is *not* the time reversal of figure 6. It is in fact an equally reversible situation. It seems to be ruled out by the properties of the absorber atoms, which are nearly all in their ground states. A fuller discussion of these topics can be found in Davies (1971b).

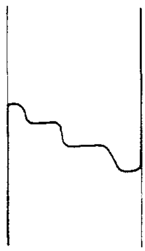

Figure 7.

Consider the case of two identical atoms well separated, one of them excited, the other in the ground state. Then we expect that the energy be transferred from one atom to the other in a characteristic time. The situation is perfectly reversible though, because there is only one initial and one final state. The energy will just oscillate between the two atoms (see, for example, Feynman and Hibbs 1965—we ignore the possibility that the atoms may radiate). If we replace the acceptor atom by a large number of similar systems so that we have a narrow band of final state energies, then we expect an irreversible transfer of energy to the group of final states. If the donor and acceptor systems are well separated we should regard the process as emission of a real photon. This is how we account for radiation in Wheeler–Feynman theory. Now the phases of the real and imaginary parts of the photon propagator D_F are correlated as described in § 1. Moreover, all frequencies k are systematically phased with respect to each other. Thus, for a two particle system, when we take the probability $|\langle S\rangle|^2$, we will have interference terms between differing frequencies k. This is the usual situation with scattering. However, in the many particle situation described above, each two particle interaction will have its own phase. When we work out the transition probability, we have to add together the amplitudes that the system will make a transition to all final states with energies in our narrow band. But each frequency k in the band corresponds to a different two particle interaction (ie a different acceptor atom) and will be randomly phased with respect to each other. Hence there are no cross terms in the summation when we take $|\langle S\rangle|^2$. That is, we may just add together probabilities rather than amplitudes. This is the usual situation with emission processes, but in the Maxwell theory the field oscillators are randomized as a separate assumption.

In quantum mechanics, obtaining a probability corresponds to making a measurement. The above analysis then suggests that the irreversibility of quantum measurement theory is closely connected with the thermodynamical and cosmological situation through the absorber. A careful examination of the connection between the microscopic quantum system, and the irreversible cosmological system in this respect might lead to a better understanding of some of the logical difficulties at the foundation of quantum mechanics and measurement theory.

5. Conclusion

Define the S matrix in the interaction picture to be

$$S = P\,\mathrm{e}^{-\mathrm{i}J} \tag{27}$$

where J is the action operator of the interaction. Then conventional quantum electrodynamics (QED) asserts

$$J = \sum_i \int j^{\mu}_{(i)}(x) A_{\mu}(x)\,\mathrm{d}x. \tag{28}$$

We have shown that the hermitean expression (28) may be replaced by the nonhermitean expression

$$-\tfrac{1}{2}\sum_i \sum_j \int\int j^{\mu}_{(i)}(x) D_{\mathrm{F}}(x-y) j_{(j)\mu}(y)\,\mathrm{d}x\,\mathrm{d}y \tag{29}$$

without change in the results for internal (virtual) photons. We may also use (29) to recover the matrix elements for real photon processes, such as emission, when we take into account an ultimate source and absorber of the radiation. All these results were proved by Feynman (1950) and have now been verified in S matrix theory. In this picture we could do away with the photons altogether if it were not for the pole in the propagator of D_{F}. This is an expression of the fact that we can still have a real (though internal) photon, and hence (as can be seen from unitarity considerations) the system is capable of emitting a real photon. Physically, this is because we cannot be sure that all photons emitted are subsequently absorbed unless we enclose the system in a light tight box. If this is done and all particles in the box are included in the matrix elements, we find that the pole in D_{F} is removed by cancellation of contributions from the box. We can therefore replace D_{F} with $\bar{D}$. That is, we may still describe all QED processes correctly, but with truly virtual ($k^2 \neq 0$) photons only. However, virtual photons are coherent, and always tied by the source currents. They may therefore be eliminated as an auxiliary concept, and we arrive at a truly direct intercurrent action theory with no photons, mediated by the $\bar{D}$ Green function. This then represents the quantum generalization of the classical Wheeler–Feynman absorber theory, although it is puzzling that S is not unitary for all fermion states in this case (it is, of course, unitary for the light tight box).

The question of whether or not the universe as a whole behaves like a light tight box is debatable, and a question of cosmology. Should this not prove the case, then the work is still of some academic interest since it tells us something of the structure of QED.

The original motivation for a direct interparticle action theory of electrodynamics was the elimination of the selfenergy divergences which plague the conventional theory. It does not seem possible to quantize the divergent free classical theory in a meaningful way without re-introducing a form of selfaction. However, with the photon propagator now relegated to the role of a potential, we are free to modify it without deriving the modification from the dynamics of the Maxwell field. Progress in removing divergences has recently been made by Sudarshan (1971) and Hoyle and Narlikar (1971).

Perhaps the most useful outcome of this work is a clearer understanding of the way in which the nature of real and virtual photons is interwoven with the distant absorber. If we accept the full Wheeler–Feynman philosophy, we would take the following attitude towards QED. That the cosmological structure of the universe allows us to add a pole to the Green function $\bar{D}$ permitting it to be interpreted as a propagator of independent mechanical particles called photons. Even if we do not accept this in full, the theory perhaps contributes to our understanding of the ways in which the large scale properties of the universe can affect the structure of local physical laws.

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References

Akhiezer A I and Berestetskii V B 1965 *Quantum Electrodynamics* (New York: Interscience)
Davies P C W 1971a *J. Phys.* A: *Gen. Phys.* **4** 836–45
—— 1971b *Phys. Bull.* **22** 211–4
Dirac P A M 1938 *Proc. R. Soc.* A **167** 148–68
Feynman R P 1950 *Phys. Rev.* **80** 440–57
Feynman R P and Hibbs A R 1965 *Quantum Mechanics and Path Integrals* (New York: McGraw-Hill)
Hoyle F and Narlikar J V 1969 *Ann. Phys.* **54** 207–39
Sudarshan E C G 1971 *University of Texas Preprint* CPT 55-AEC-14
Wheeler J A and Feynman R P 1945 *Rev. mod. Phys.* **17** 157–81